The Paretian Optimum

Pareto Optimality in Production and Perfect Competition:

Pareto optimality in production is guaranteed under perfect competition. For, under perfect competition, the prices r_1 and r_2 of the two inputs, X_1 and X_2, are given to the firms that produce the goods Q_1 and Q_2, and each profit-maximising firm equates the $MRTS_{X1,x2}$ to the ratio of the prices of the inputs.

That is, for the producer of Q_1 we get:

$$\frac{\frac{\partial q_1}{\partial x_{11}}}{\frac{\partial q_1}{\partial x_{12}}} = \frac{r_1}{r_2}$$

and for the producer of good Q_2, we get (21.8)

$$\frac{\frac{\partial q_2}{\partial x_{21}}}{\frac{\partial q_2}{\partial x_{22}}} = \frac{r_1}{r_2}$$

From (21.8), we obtain:

$MRTS_{x1,\,x2}$ in the production of Q_1 = $MRTS_{x1,x2}$ in the production of Q_2 (21.9)

Since condition (21.9) is the same as condition (21.7), Pareto efficiency in production is a certainty under perfect competition.

We may now obtain a graphical solution of equation (21.7) or (21.9) for the allocation of inputs X_1 and X_2 over the production of goods Q_1 and Q_2 and for the quantities produced of Q_1 and Q_2. The satisfaction of the marginal condition (21.7) or (21.9) is guaranteed under perfect competition.

Let us suppose that in the competitive markets the prices of the inputs are given to be r_1 and r_2. Let us now draw a straight line ST of slope $-r_1/r_2$ through the point O' in Fig. 21.1, and pick up the point e on the contract curve for production (CCP) where the common slope of the isoquants has been equal to the slope of the line ST. That is, at the point e, we have numerical slopes of the IQs of two individuals = the numerical slope of the line ST = r_1/r_2

$$\Rightarrow MRTS^{Q_1}_{X_1,X_2} = MRTS^{Q_2}_{X_1,X_2} = \frac{r_1}{r_2} \qquad (21.10)$$

That is, at the point e in Fig. 21.1, the marginal condition for efficiency of production has been satisfied. At this point quantities of the two inputs, x^0_{11} and x^0_{21} would be used in the production of Q_1 and these quantities, when substituted in the production function for Q_1, would give us the output quantity of Similarly, quantities of the two inputs, x^0_{21} and x^0_{22}, would be used in the production of Q_2 and the output here would be q^0_2